MANSOURA UNIVERSITY
FACULTY OF ENGINEERING
DEPARTMENT OF ELECTRONICS
& ELECTRICAL COMMUNICATIONS

ELECTROMAGNETIC FIELDS
SECOND YEAR
EXAM JAN 2010
Time 3 Hours

Attempt the following questions. Full mark 100

Q1: a- Show that the potential V of an electric dipole placed at the origin is $V = \dfrac{\overline{P} \bullet \hat{r}}{4\pi\varepsilon r^2}$.

b- Derive expression for the electric field .

c- Determine V if the dipole is centered at r'.

d- An electric dipole $\overline{P} = 0.10\,\hat{z}\,\mu C\, m$ is located at the point (0,0,2) in free space and the plane z = 0 is perfectly conducting. Use the method of images to find V at the point P(0,0,1).

Marks 20

Q2: a- The surface S separates two non-conducting regions (μ_1, ε_1) and (μ_2, ε_2). Apply Gauss laws to a suitable Gaussian surface to derive the boundary conditions on the normal components of B and D at the interface S.

b- The half space $z > 0$ is air while the region $z < 0$ is filled with a ferrite material for which $\chi_m = 2.5$ and $\chi_e = 7.5$. If $H = 0.5\,\hat{y} + 0.5\,\hat{z}$ mA/m and $E = -0.188\,\hat{y} + 0.188\,\hat{z}$ V/m in air, find H and D in the ferrite material.

c- Find the polarization P, the bound surface charge density , the magnetization M, and the bound surface current density in the region z < 0.

d- What are the densities of the electric and magnetic energies in each region?

Marks 20

Q3: a- Write Laplace's equation in cylindrical coordinates (ρ, φ,z).

b- A coaxial line has an inner conductor of radius a and an outer conductor of radius b The region between the conductors is filled with a dielectric of permittivity ε F/m. The inner conductor is at voltage U and the outer is earthed (V = 0). By solving Laplace's equation, determine the potential and electric field distributions between conductors. Show that the capacitance per unit length of the line is $C = 2\pi\varepsilon / \ln(b/a)$.

c- Deduce an expression for the inductance per unit length.

d- If the dielectric has a small conductivity σ S/m, deduce the leakage conductance per unit length.

e- If the voltage U is sinusoidal: $U = U_o \sin \omega t$, show that the ratio between the conduction current and the displacement currents through the dielectric is $\sigma / \omega\varepsilon$.

Marks 22

Q4: A long copper wire of radius a carries a total current I. The wire is along the z axis and the current is uniformly distributed on the cross-section.

a- Find the current density J.

b- Use Ampere's law and suitable amperian paths to obtain expressions for the magnetic field H inside and outside the wire.

c- Use energy relations to show that the internal inductance per unit length of the wire is $\mu_o / 8\pi$ H/m.

Marks 18

47

Q5: a- The electrostatic force is a conservative force. Is this still true when we allow time variation of the fields? If not, does this violate the principal of conservation of energy?
b- Is the magnetic field a conservative field? Why?
c- A charge q enters the region of parallel electric and magnetic fields with zero initial velocity. What is the shape of the path?
d- Two parallel wires carry the equal currents in the same direction. Is the force acting on them attractive or repulsive?
e- Define the vector magnetic potential A. Using this definition, Amperes law, and Coulomb's gauge, show that $\bar{A} = \frac{\mu}{4\pi}\int \frac{\bar{J}\, dvol}{r}$.
f- A conducting body having a smooth surface carries a total charge Q and is at potential U. A spherical cavity of radius r is made somewhere inside the body. What is the charge density on the surface of the cavity?
g- The potential at points inside the cavity in the last question is U. Show that the uniqueness theorem implies this result.
h- Polarization and magnetization are due to electric and magnetic dipoles already existing in the material or induced in it by external fields. What are the main mechanisms of polarization and magnetization?
g- By taking the divergence of the second curl equation of Maxwell, derive the continuity equation: div J + $\varepsilon\frac{\partial \rho}{\partial t} = 0$.
i- The potential V satisfies Laplace's equation inside a region R and certain boundary conditions on its boundaries. Explain briefly why V can not have maximum or minimum values inside R and as a consequence, it is impossible to hold a charge in stable equilibrium with electrostatic fields (Earnshaw's theorem).

Marks 30
